DAY THIRTY SIX

Mathematical Reasoning

Learning & Revision for the Day

- Statement (Proposition)
- Truth Value and Truth Table
- Elementary Operations of Logic
- Converse, Inverse and Contrapositive of an Implication
- Tautology
- Contradiction (Fallacy)
- Algebra of Statement

Sentence

A sentence is a relatively independent grammatical unit. It can stand alone or it can be combined with other sentences to form a text, a story etc. Sentences can be divided into different types as declarative, interrogative, imperative, exclamatory and operative sentences.

Statement (Proposition)

A statement is a sentence which is either true or false but not both simultaneously i.e. ambiguous sentence are not considered as statements.

The working nature of statement in logic is same as nature of switch in circuit.

i.e. Switch $\begin{cases} \text{ON (1)} \\ \text{OFF (0)} \end{cases}$ and Statement $\begin{cases} \text{True (T)} \\ \text{False (F)} \end{cases}$

Types of statements

1. **Simple statement** A statement, which cannot be broken into two or more sentences, is a simple statement.

   Generally, small letters $p, q, r, \ldots$ denote simple statements.
2. **Compound statement** A statement formed by two or more simple statements using the words such as "and", "or", "not", "if then", "if and only if", is called a compound statement.

3. **Substatements** Simple statements which when combined to form a compound statement are called substatements or components.

NOTE
- A true statement is known as a valid statement.
- A false statement is known as an invalid statement.
- Imperative, exclamatory, interrogative, optative sentences are not statements.
- Mathematical identities are considered to be statements because they can either be true or false but not both.

4. **Open statement** A sentence which contains one or more variables such that when certain values are given to the variable it becomes a statement, is called an open statement.

Truth Value and Truth Table

- A statement can be either 'true' or 'false', which are called truth values of a statement and it is represented by the symbols 'T' and 'F', respectively.
- A table that shows the relationship between the truth values of compound statement, $S(p, q, r, \dots)$ and the truth values of its substatements $p, q, r, \dots$ etc., is called the truth table of statement S.
- If a compound statement has simply n substatements, then there are 2^n rows representing logical possibilities.

Logical Connectives or Sentencial Connectives

Two or more statements are combined to form a compound statement by using symbols. These symbols are called logical connectives.

Logical connectives are given below

Words	Symbols
and	$\wedge$
or	$\vee$
implies that (if ..., then)	$\Rightarrow$
If and only if (implies and is implied by)	$\Leftrightarrow$

Elementary Operations of Logic

Formation of compound sentences from simple sentence using logical connectives are termed as elementary operation of logic.

There are five such operations, which are discussed below.

1. Negation (Inversion) of Statement

(i) A statement which is formed by changing the truth value of a given statement by using word like 'no' or 'not' is called negation of a given statement. It represents the symbol '$\sim$'.

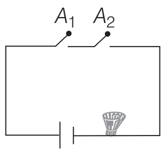
(ii) If p is statement, then negation of p is denoted by '$\sim p$'.

(iii) The truth table for NOT is given by

p	$\sim p$
T	F
F	T

2. Conjunction (AND)

A compound sentence formed by two simple sentences p and q using connective "AND" is called the conjunction of p and q and is represented by $p \wedge q$.



The truth table for operation 'AND' is given by

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

NOTE
- The statement $p \wedge q$ is true, if both p and q are true.
- The statement $p \wedge q$ is false, if atleast one of p and q or both are false.

3. Disjunction (OR)

A compound sentence formed by two simple sentences p and q using connective "OR" is called the disjunction of p and q and is represented by $p \vee q$.

The truth table for operation 'OR' is given by

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

NOTE
- The statement $p \vee q$ is true, if atleast one of p and q or both are true.
- The statement $p \vee q$ is false, if both p and q are false.

4. Implication (Conditional)

A compound sentence formed by two simple sentences p and q using connective "if ... then ..." is called the implication of p and q and represented by $p \Rightarrow q$ which is read as "p implies q".

Here, p is called antecedent or hypothesis and q is called consequent or conclusion.

The truth table for if … then is given by

p	q	$p \Rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

It is clear from the truth table that column III is equal to column V. i.e. statement $p \Rightarrow q$ is equivalent to $\sim p \vee q$.

5. Biconditional Statement

Two simple sentences connected by the phrase "if and only if," form a biconditional statement. It is represented by the symbol '$\Leftrightarrow$'.

The truth table for if and only if is given by

p	q	$p \Leftrightarrow q$	$\sim p$	$\sim q$	$\sim p \vee q$	$p \vee \sim q$	$(\sim p \vee q) \wedge (p \vee \sim q)$
T	T	T	F	F	T	T	T
T	F	F	F	T	F	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T

NOTE
- It is clear from the truth table that column III is equal to column VIII. i.e. statement $p \Leftrightarrow q$ is equivalent to $(\sim p \vee q) \wedge (p \vee \sim q)$.
- The statement $p \Leftrightarrow q$ is true, if either both are true or both are false.
- The statement $p \Leftrightarrow q$ is false, if exactly one of them is false.

Converse, Inverse and Contrapositive of an Implication

For any two statements p and q,

(i) Converse of the implication 'if p, then q' is 'if q, then p' i.e. $q \Rightarrow p$

(ii) Inverse of the implication 'if p, then q' is 'if $\sim p$, then $\sim q$' i.e. $\sim p \rightarrow \sim q$.

(iii) Contrapositive of the implication 'if p, then q' is 'if $\sim q$, then $\sim p$' i.e. $\sim q \rightarrow \sim p$.

NOTE
- $\sim (p \Rightarrow q) \equiv \sim (\sim p \vee q) = \{p \wedge (\sim q)\}$ $\therefore$ $\sim (p \Leftrightarrow q) \equiv \{p \wedge (\sim q)\}$
- $\sim (p \Rightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$
- $p \Rightarrow q = \sim p \vee q$
- $(p \Leftrightarrow q) \Leftrightarrow r = p \Leftrightarrow (q \Leftrightarrow r)$
- $p \Leftrightarrow q = (p \Rightarrow q) \wedge (q \Rightarrow p)$

Tautology

A compound statement is called a tautology, if it has truth value T whatever may be the truth value of its compounds.

Statement $(p \Rightarrow q) \wedge p \Rightarrow q$ is a tautology.

The truth table of above statement is prepared as follows.

p	q	$p \Rightarrow q$	$p \Rightarrow q \wedge p$	$(p \Rightarrow q) \wedge p \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Contradiction (Fallacy)

A compound statement is called contradiction, if its truth value is F whatever may be the truth value of its components.

Statement $\sim p \wedge p$ is a contradiction.

The truth table of above statement is prepared as follows

p	$\sim p$	$\sim p \wedge p$
T	F	F
F	T	F

A statement which is neither a tautology nor a contradiction is a contigency.

Algebra of Statement

Some of the important laws considered under the category of algebra of statements are given as

1. **Idempotent Laws**
   For any statement p, we have
   (a) $p \wedge p \equiv p$ (b) $p \vee p \equiv p$
2. **Commutative Laws**
   For any two statements p and q, we have
   (a) $p \wedge q \equiv q \wedge p$ (b) $p \vee q \equiv q \vee p$
3. **Associative Laws**
   For any three statements p, q and r, we have
   (a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
   (b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$
4. **Distributive Laws**
   (a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
   (b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
5. **Involution Laws**
   For any statement p, we have $\sim (\sim p) \equiv p$
6. **De-morgan's Laws**
   (a) $\sim (p \wedge q) = \sim p \vee \sim q$
   (b) $\sim (p \vee q) = \sim p \wedge \sim q$

7. **Complement Laws**
For any statement p, we have
(a) $p \vee \sim p \equiv T$ (b) $p \wedge \sim p \equiv F$
(c) $\sim T \equiv F$ (d) $\sim F \equiv T$

8. **Identity Laws**
For any statement p, we have
(a) $p \wedge T \equiv p$ (b) $p \vee F \equiv p$
(c) $p \vee T \equiv T$ (d) $p \wedge F \equiv F$
where, T and F are the true and false statement.

9. **Duality**
- Two compound statements S_1 and S_2 are said to be duals of each other, if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.
- The connectives $\wedge$ and $\vee$ are also called duals of each other.
- Symbolically, it can be written as, if $S(p, q) = p \wedge q$, then its dual is $S^*(p, q) = p \vee q$.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 Which of the following is not a statement? ➔ **NCERT Exemplar**
(a) Smoking is injurious to health
(b) 2 + 2 = 4
(c) 2 is the only even prime number
(d) Come here

2 The negation of the statement "72 is divisible by 2 and 3" is ➔ **NCERT Exemplar**
(a) 72 is not divisible by 2 or 72 is not divisible by 3
(b) 72 is not divisible by 2 and 72 is not divisible by 3
(c) 72 is divisible by 2 and 72 is not divisible by 3
(d) 72 is not divisible by 2 and 72 is divisible by 3

3 The dual of the statement $(p \vee q) \vee r$ is
(a) $(p \wedge q) \vee r$ (b) $(p \wedge q) \wedge r$
(c) $(p \vee q) \wedge r$ (d) None of these

4 Choose disjunction among the following sentences
(a) It is raining and the Sun is shining
(b) Ram and Shyam are good friends
(c) 2 or 3 is a prime number
(d) Everyone who lies in India is an Indian

5 Which among the following is not a conjunction?
(a) Gautam and Rahul are good friends
(b) The Earth is round and the Sun is hot
(c) 9> 4 and 12 > 15
(d) None of the above

6 If p : a natural number n is odd and q : natural number n is not divisible by 2, then the biconditional statement $p \Leftrightarrow q$ is ➔ **NCERT**
(a) A natural number n is odd if and only if it is divisible by 2
(b) A natural number n is odd if and only if it is not divisible by 2
(c) If a natural number n is odd, then it is not divisible by 2
(d) None of the above

7 The logically equivalent proposition of $p \Leftrightarrow q$ is
(a) $(p \wedge q) \vee (p \wedge q)$ (b) $(p \Rightarrow q) \wedge (q \Rightarrow p)$
(c) $(p \wedge q) \vee (q \Rightarrow p)$ (d) $(p \wedge q) \Rightarrow (p \vee q)$

8 If both p and q are false, then
(a) $p \wedge q$ is true (b) $p \vee q$ is true
(c) $p \Rightarrow q$ is true (d) $p \Leftrightarrow q$ is false

9 The negation of '12 > 4' is
(a) $12 \le 4$ (b) $13 > 5$ (c) $12 > 3$ (d) $12 \ge 4$

10 Which among the following is not a negation of 'The Sun is a star'?
(a) The Sun is not a star.
(b) It is not the case that Sun is a star.
(c) It is false that the Sun is a star.
(d) It is true that the Sun is a star.

11 Which of the following is not equivalent to $p \leftrightarrow q$?
(a) p if and only if q
(b) p is necessary and sufficient for q
(c) q if and only if p
(d) None of the above

12 For integers m and n, both greater than 1, consider the following three statements ➔ **JEE Mains 2013**
$P : m$ divides n $Q : m$ divides n^2 $R : m$ is prime, then
(a) $Q \wedge R \to P$ (b) $P \wedge Q \to R$
(c) $Q \to R$ (d) $Q \to P$

13 If p and q are two statements such that
p : the questions paper is easy
q : we shall pass,
then the symbolic statement $\sim p \to \sim q$ means
(a) If the question paper is easy, then we shall pass
(b) If the question paper is not easy, then we shall not pass
(c) The question paper is easy and we shall pass
(d) The question paper is easy or we shall pass

14 For the following three statements

p : 2 is an even number

q : 2 is a prime number.

r : Sum of two prime numbers is always even, then the symbolic statement $(p \wedge q) \rightarrow \sim r$ means

(a) 2 is an even and prime number and the sum of two prime numbers is always even
(b) 2 is an even and prime number and the sum of two prime numbers is not always even
(c) 2 is an even and prime number, then the sum of two prime numbers is not always even
(d) 2 is an even and prime number, then the sum of two prime numbers is always even

15 The converse of the statement

"If $x > y$, then $x + a > y + a$" is → **NCERT Exemplar**

(a) If $x < y$, then $x + a < y + a$ (b) If $x + a > y + a$, then $x > y$
(c) If $x < y$, then $x + a > y + a$ (d) If $x > y$, then $x + a < y + a$

16 Consider the following statements

p : If a number is divisible by 10, then it is divisible by 5.

q : If a number is divisible by 5, then it is divisible by 10.

Then, the correct option is

(a) q is converse of p (b) p is converse of q
(c) p is not converse of q (d) Both (a) and (b)

17 The negation of the statement. "If I become a teacher, then I will open a school", is → **AIEEE 2012**

(a) I will become a teacher and I will not open a school
(b) Either I will not become a teacher or I will not open a school
(c) Neither I will become a teacher nor I will open a school
(d) I will not become a teacher or I will open a school

18 Find the contrapositive of "If two triangles are identical, then these are similar".

(a) If two triangles are not similar, then these are not identical
(b) If two triangles are not identical, then these are not similar
(c) If two triangles are not identical, then these are similar
(d) If two triangles are not similar, then these are identical

19 The contrapositive of the statement

"If 7 is greater than 5, then 8 is greater than 6" is

→ **NCERT Exemplar**

(a) If 8 is greater than 6, then 7 is greater than 5
(b) If 8 is not greater than 6, then 7 is greater than 5
(c) If 8 is not greater than 6, then 7 is not greater than 5
(d) If 8 is greater than 6,then 7 is not greater than 5

20 The conditional statement of

"You will get a sweet dish after the dinner" is

→ **NCERT Exemplar**

(a) If you take the dinner, then you will get a sweet dish
(b) If you take the dinner, you will get a sweet dish
(c) You get a sweet dish if and only if you take the dinner
(d) None of the above

21 Let S be a non-empty subset of R. Consider the following statement

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P? → **AIEEE 2010**

(a) There is a rational number $x \in S$ such that $x \le 0$
(b) There is no rational number $x \in S$ such that $x \le 0$
(c) Every rational number $x \in S$ satisfies $x \le 0$
(d) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational

22 The contra positive of $p \rightarrow (\sim q \rightarrow \sim r)$ is

(a) $(\sim q \wedge r) \rightarrow \sim p$ (b) $(q \wedge \sim r) \rightarrow \sim p$
(c) $p \rightarrow (\sim r \vee q)$ (d) $p \wedge (q \wedge r)$

23 $p \vee \sim (p \wedge q)$ is a

(a) contradiction (b) contingency
(c) tautology (d) None of these

24 If p, q and r are simple propositions with truth values T, F, T respectively, then the truth value of $(\sim p \vee q) \wedge \sim q \rightarrow p$ is

(a) true (b) false
(c) true, if r is false (d) None of these

25 The statement $p \rightarrow (q \rightarrow p)$ is equivalent to → **JEE Mains 2013**

(a) $p \rightarrow q$ (b) $p \rightarrow (p \vee q)$
(c) $p \rightarrow (p \rightarrow q)$ (d) $p \rightarrow (p \wedge q)$

26 The statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is → **JEE Mains 2017**

(a) a tautology
(b) equivalent to $\sim p \rightarrow q$
(c) equivalent to $p \rightarrow \sim q$
(d) a fallacy

27 Let p and q be two statements. Then, $(\sim p \vee q) \wedge (\sim p \wedge \sim q)$ is a

(a) tautology
(b) contradiction
(c) neither tautology nor contradiction
(d) both tautology and contradiction

28 The proposition $(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$ is

(a) contigency
(b) neither tautology nor contradiction
(c) contradiction
(d) tautology

29 If p and q are two statements, then

$$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p) \text{ is a}$$

(a) contradiction (b) tautology
(c) neither (a) nor (b) (d) None of these

30 The proposition $S : (p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$ is

(a) a tautology (b) a contradiction
(c) either (a) or (b) (d) neither (a) nor (b)

31 The statement $\sim(p \leftrightarrow \sim q)$ is

(a) equivalent to $p \leftrightarrow q$ (b) equivalent to $\sim p \leftrightarrow q$
(c) a tautology (d) a fallacy

32 The false statement in the following is

(a) $p \wedge (\sim p)$ is a contradiction
(b) $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a contradiction
(c) $\sim(\sim p) \leftrightarrow p$ is a tautology
(d) $p \vee (\sim p)$ is a tautology

33 Among the following statements, which is a tautology?

(a) $p \wedge (p \vee q)$ (b) $p \vee (p \wedge q)$
(c) $[p \wedge (p \to q)] \to q$ (d) $q \to [p \wedge (p \to q)]$

34 Consider the following statements

P : Suman is brilliant.

Q : Suman is rich.

R : Suman is honest.

The negative of the statement.'Suman is brilliant and dishonest if and only if Suman is rich.' can be expressed as **➜ AIEEE 2011**

(a) $\sim(Q \leftrightarrow (P \wedge \sim R))$ (b) $\sim(Q \leftrightarrow P \wedge R)$
(c) $\sim(P \wedge \sim R) \leftrightarrow Q$ (d) $\sim P \wedge (Q \leftrightarrow \sim R)$

35 The statement $(p \Rightarrow q) \Leftrightarrow (\sim p \wedge q)$ is a

(a) tautology (b) contradiction
(c) neither (a) nor (b) (d) None of these

36 The proposition $\sim(p \Rightarrow q) \Rightarrow (\sim p \vee \sim q)$ is

(a) a tautology (b) a contradiction
(c) either (a) or (b) (d) neither (a) nor (b)

37 The negation of the compound proposition is $p \vee (\sim p \vee q)$

(a) $(p \wedge \sim q) \wedge \sim p$ (b) $(p \vee \sim q) \vee \sim p$
(c) $(p \wedge \sim q) \vee \sim p$ (d) None of these

38 The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to **➜ JEE Mains 2015**

(a) $s \wedge \sim r$ (b) $s \wedge (r \wedge \sim s)$
(c) $s \vee (r \vee \sim s)$ (d) $s \wedge r$

39 $\sim S(p, q)$ is equivalent to

(a) $S^*(\sim p, \sim q)$ (b) $S^*(p, \sim q)$
(c) $S^*(\sim p, q)$ (d) None of these

40 Let p : 25 is a multiple of 5.

q : 25 is a multiple of 8.

Statement I The compound statement "p and q" is false.

Statement II The compound statement "p or q" is false.

Choose the correct option

(a) Only Statement I is correct
(b) Only Statement II is correct
(c) Both statements are correct
(d) Both statements are incorrect

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 If p : 4 is an even prime number, q : 6 is divisor of 12 and r : the HCF of 4 and 6 is 2, then which one of the following is true?

(a) $(p \wedge q)$ (b) $(p \vee q) \wedge \sim r$
(c) $\sim(q \wedge r) \vee p$ (d) $\sim p \vee (q \wedge r)$

2 An equivalent expression for $(p \Rightarrow q \wedge r) \vee (r \Leftrightarrow s)$ which contains neither the biconditional nor the conditional is

(a) $(\sim p \vee q \wedge r) \vee ((\sim r \vee s) \wedge (r \vee \sim s))$
(b) $(\sim p \wedge q \wedge r) \vee ((\sim r \vee s) \wedge (r \vee \sim s))$
(c) $(\sim p \vee q \wedge r) \wedge ((\sim r \vee s) \vee (r \vee \sim s))$
(d) None of the above

3 If $(p \wedge \sim r) \Rightarrow (\sim p \vee q)$ is false, then the truth values of p, q and r respectively

(a) T, F and F (b) F, F and T
(c) F, T and T (d) T, F and T

4 The Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to **➜ JEE Mains 2016**

(a) $\sim p \wedge q$ (b) $p \wedge q$
(c) $p \vee q$ (d) $p \vee \sim q$

5 Which of the following is a tautology?

(a) $(p \to q) \wedge (p \to q)$ (b) $(p \to q) \vee (p \to q)$
(c) $(p \to q) \vee (q \to p)$ (d) None of these

6 Let p and q stand for the statements '$2 \times 4 = 8$' and '4 divides 7', respectively. Then, what are the truth values for following biconditional statements?

(i) $p \leftrightarrow q$ (ii) $\sim p \leftrightarrow q$
(iii) $\sim q \leftrightarrow p$ (iv) $\sim p \leftrightarrow \sim q$

(a) T T T T (b) F T T T
(c) F T F F (d) F T T F

7 The only statement among the followings that is a tautology is **➜ AIEEE 2011**

(a) $B \to [A \wedge (A \to B)]$ (b) $A \wedge (A \vee B)$
(c) $A \vee (A \wedge B)$ (d) $[A \wedge (A \to B)] \to B$

8 Which of the following is not correct?

(a) $\sim(p \wedge q) = (\sim p) \vee (\sim q)$
(b) Truth value of $p \wedge q$ = truth value of $q \wedge p$
(c) $\sim(\sim p) = p$
(d) $p \Leftrightarrow q \equiv (p \Rightarrow q) \vee (q \Rightarrow p)$

9 $\sim(p \Rightarrow q) \Leftrightarrow \sim p \vee \sim q$ is
(a) a tautology (b) a contradiction
(c) neither a tautology nor a contradiction
(d) cannot come to any conclusion

10 Which of the following is wrong statement?
(a) $p \rightarrow q$ is logically equivalent to $\sim p \vee q$
(b) If the truth values of p, q, r are T, F, T respectively, then the truth value of $(p \vee q) \wedge (q \vee r)$ is T
(c) $\sim (\vee q \vee q \vee r) \cong \sim p \wedge \sim q \wedge \sim r$
(d) The truth value of $p \wedge \sim (p \vee q)$ is always T

11 If p, q and r are simple propositions, then $(p \wedge q) \wedge (q \wedge r)$ is true, then
(a) p, q and r are true (b) p, q are true and r is false
(c) p is true and q, r are false (d) p, q and r are false

12 $\sim p \wedge q$ is logically equivalent to
(a) $p \Rightarrow q$ (b) $q \Rightarrow p$
(c) $\sim (p \Rightarrow q)$ (d) $\sim (q \Rightarrow p)$

13 Which of the following is true for any two statements p and q?
(a) $\sim [p \vee \sim q] \equiv \sim p \wedge q$ (b) $\sim p \wedge q$ is a fallacy
(c) $p \vee \sim q$ is a tautology (d) $p \vee \sim p$ is a contradiction

14 Consider

Statement I $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement II $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow p)$ is a tautology.

➜ JEE Mains 2013

(a) Statement I is true; Statement II is true; Statement II is a correct explanation for Statement I
(b) Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

15 **Statement I** The statement $A \rightarrow (B \rightarrow A)$ is equivalent to $A \rightarrow (A \vee B)$.

Statement II The statement $\sim \{(A \wedge B) \rightarrow (\sim A \vee B)\}$ is tautology. **➜ JEE Mains 2013**

(a) Statement I is true, Statement II is false
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I
(d) Statement I is false, Statement II is true

ANSWERS

SESSION 1	**1** (d)	**2** (a)	**3** (b)	**4** (c)	**5** (a)	**6** (b)	**7** (b)	**8** (c)	**9** (a)	**10** (d)
	11 (d)	**12** (a)	**13** (b)	**14** (c)	**15** (b)	**16** (d)	**17** (a)	**18** (a)	**19** (c)	**20** (a)
	21 (c)	**22** (a)	**23** (c)	**24** (a)	**25** (b)	**26** (a)	**27** (c)	**28** (c)	**29** (b)	**30** (a)
	31 (a)	**32** (b)	**33** (c)	**34** (a)	**35** (c)	**36** (a)	**37** (a)	**38** (d)	**39** (a)	**40** (a)
SESSION 2	**1** (d)	**2** (a)	**3** (a)	**4** (c)	**5** (c)	**6** (d)	**7** (d)	**8** (d)	**9** (c)	**10** (d)
	11 (a)	**12** (d)	**13** (a)	**14** (b)	**15** (c)					

Hints and Explanations

SESSION 1

1 In option (a), (b) and (c).
It is a declarative sentence, which is clearly true, therefore it is a true statement.
In option (d), it is an imperative sentence, therefore it is not a statement.

2 The negation of the given statement is "72 is not divisible by 2 or 72 is not divisible by 3".

3 According to definition let $s(p,q) = p \wedge q$ be a compound statement. Then, $s^*(p,q) = p \vee q$,
Therefore, the dual of $(p \vee q) \vee r$ is $(p \wedge q) \wedge r$.

4 In disjunction, two sentences are connected with 'OR'.

5 In the sentence, 'Gautam and Rahul are good friends', the word 'and' is not a connective. So, this sentence is not a conjunction.

6 Given, p : A natural number n is odd and q : natural number n is not divisible by 2. The biconditional statement $p \Leftrightarrow q$ i.e. "A natural number n is odd if and only if it is not divisible by 2".

7 $(p \Rightarrow q) \wedge (q \Rightarrow p)$ means $p \Leftrightarrow q$. Hence, option (b) is correct.

8 If both p and q are false, then $p \Rightarrow q$ is true.

9 Negation is a denial of a statement.
So, $12 \leq 4$ is correct option.

10 The denial of any statement is called its negation.
'It is true that the Sun is a star' is an assertion of the given statement.

11 Options (a), (b) and (c) are equivalent to $p \leftrightarrow q$.

12 $P : \frac{n}{m}; Q : \frac{n^2}{m}; R : m$ is prime.

Let $m = 5, p = 10, n^2 = 100, m = 3, n = 12, n^2 = 144$

$m = 7, n = 14, n^2 = 196$

So, P, Q and R are true statements.

$\therefore \quad Q \wedge R = \mathrm{T} \wedge \mathrm{T} \to \mathrm{T} = P$

13 Given, p : The question paper is easy and q : We shall pass given.
The symbolic representation of given option are
(a) : $p \to q$ (b) : $\sim p \to \sim q$ (c) : $p \wedge q$ (d) : $p \vee q$

14 Given, p : 2 is an even number
q : 2 is a prime number
r : sum of two prime numbers is always even.
The symbolic representation of given option (s) are
(a) : $p \wedge q \wedge r$ (b) : $p \wedge q \wedge \sim r$
(c) : $p \wedge q \Rightarrow \sim r$ (d) : $p \wedge q \Rightarrow r$

15 Converse statement is "If $x + a > y + a$, then $x > y$".

16 The converse of the statement "if p then q" is "if q then p".
Given statements are
p : if a number is divisible by 10, it is divisible by 5.
q : if a number is divisible by 5, then it is divisible by 10.
It is clear that q is converse of p and p is converse of q.

17 Let us assume that p : 'I become a teacher' and q : I will open a school. Then, we can easily ascertain that
Negation of $(p \to q)$ is $\sim (p \to q) = p \wedge \sim q$, which means that 'I will become a teacher and I will not open a school.'

18 Consider the following statements
p : Two triangles are identical.
q : Two triangles are similar.
Clearly, the given statement in symbolic form is $p \Rightarrow q$.
So, its contrapositive is given by $\sim q \Rightarrow \sim p$.

19 The contrapositive of the given statement is "If 8 is not greater than 6, then 7 is not greater than 5.

20 The conditional statement of given statement is "If you take the dinner, then you will get a sweet dish".

21 P : There is rational number $x \in S$ such that $x > 0$.
$\sim P$: Every rational number $x \in S$ satisfies $x \leq 0$.

22 The contrapositive of $p \to q$ is $\sim q \to \sim p$
$\therefore$ Contrapositive of $p \to (\sim q \to \sim r)$ is $\sim (\sim q \to \sim r) \to \sim p$

$$\equiv (\sim q \wedge r) \to \sim p \qquad \left[\begin{array}{l} \because \sim (p \to q) \equiv p \wedge \sim q \\ \therefore \sim (\sim q \to \sim r) \equiv \sim q \wedge r \end{array}\right]$$

23

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

24

p	q	r	$\sim p$	$\sim q$	$\sim p \vee q$	$(\sim p \vee q) \wedge \sim q$	$(\sim p \vee q) \wedge \sim q \to p$
T	F	T	F	T	F	F	T

25

p	q	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	T

So, statement $p \to (q \to p)$ is logically equivalent to $p \to (p \vee q)$.

26 The truth table of the given expression is given below :

p	q	$x \equiv p \to q$	$\sim p$	$\sim p \to q$	$y \equiv (\sim p \to q) \to q$	$x \to y$
T	T	T	F	T	T	T
T	F	F	F-	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T

Hence, it is a tautology.

27

p	q	$\sim p$	$\sim q$	$(\sim p \vee q)$	$(\sim p \wedge \sim q)$	$(\sim p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Hence, it is neither tautology nor contradiction.

28

p	$\sim p$	$p \Rightarrow \sim p$	$\sim p \Rightarrow p$	$(p \Rightarrow \sim p) \wedge (\sim p \Rightarrow p)$
F	T	T	F	F
T	F	F	T	F

$\therefore$ Statement is contradiction.

29

p	q	$\sim q$	$\sim p$	$\sim q \Rightarrow \sim p$	$p \Leftrightarrow q$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

Hence, it is tautology.

30

p	q	$p \Rightarrow q$	$\sim p$	$(\sim p \vee q)$	$(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Since, all values of given proposition is true, hence it is a tautology.

31

p	q	$\sim p$	$\sim q$	$q \leftrightarrow q$	$p \leftrightarrow \sim q$	$\sim p \leftrightarrow q$	$\sim(p \leftrightarrow \sim q)$
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
T	T	F	F	T	F	F	T
F	F	T	T	T	F	F	T

$\sim(p \leftrightarrow \sim q)$ is equivalent to $(p \leftrightarrow q)$.

32 $p \to q$ is equivalent to $\sim q \to \sim p$

$\therefore \quad (p \to q) \leftrightarrow (\sim q \to \sim p)$

is a tautology but not a contradiction.

33 $p \wedge (p \vee q)$ is F, when $p \equiv$ F

$p \vee (p \wedge q)$ is F, when $p \equiv$ F, $q \equiv$ F

and $q \to [p \wedge (p \to q)]$ is F, when $p \equiv$ F, $q \equiv$ T

So, for $[p \wedge (p \to q)] \to q \equiv [p \wedge (\sim p \vee q)] \to q$

$\equiv [\{p \wedge (\sim p)\} \vee (p \wedge q)] \to q$

34 Suman is brilliant and dishonest, if and only if Suman is rich, is expressed as, $Q \leftrightarrow (P \wedge \sim R)$

So, negation of it will be $\sim(Q \leftrightarrow (P \wedge \sim R))$.

35

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \wedge q$	$(p \Rightarrow q) \Leftrightarrow (\sim p \wedge q)$
T	T	F	T	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	F	F

36 By truth table

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	$\sim p \vee \sim q$	$\sim(p \Rightarrow q) \Rightarrow (\sim p \vee \sim q)$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

Hence, given proposition is a tautology.

37 Since, $\quad S : \sim(p \vee (\sim p \vee q))$

$\Rightarrow \quad S : \sim p \wedge \sim(\sim p \vee q)$ (De-Morgan's Law)

$\Rightarrow \quad S : \sim p \wedge (p \wedge \sim q)$ (De-Morgan's law)

38 $\sim(\sim s \vee (\sim r \wedge s)) \equiv s \wedge (\sim(\sim r \wedge s)) \equiv s \wedge (r \vee \sim s)$

$\equiv (s \wedge r) \vee (s \wedge \sim s) \equiv (s \wedge r) \vee F \qquad [\because s \wedge \sim s$ is false$]$

$\equiv s \wedge r$

39 $\because \quad \sim S\,(p, q) = \sim(p \wedge q) = (\sim p) \vee (\sim q) = S^*(\sim p, \sim q)$

40 I. Compound statement with 'AND' is 25 is a multiple of 5 and 8.

This is a false statement. Since, p is true but q is false.

[since, 25 is divisible by 5 but not divisible by 8]

II. Compound statement with 'OR' is

25 is a multiple of 5 or it is a multiple of 8.

This is a true statement. Since, p is true and q is false.

SESSION 2

1 Given that, p : 4 is an even prime number.

q : 6 is a divisor of 12 and r : the HCF of 4 and 6 is 2.

So, the truth value of the statements p,q and r are F, T and T, respectively.

Hence, $\sim p \vee (q \wedge r)$ is true.

2 $(p \Rightarrow q \wedge r) \vee (r \Leftrightarrow s) \equiv (p \Rightarrow q \wedge r) \vee [(\sim r \vee s) \wedge (r \vee \sim s)]$

$\equiv (\sim p \vee q \wedge r) \vee [(\sim r \vee s) \wedge (r \vee \sim s)]$

$[\because p \Rightarrow q \wedge r \equiv \sim p \vee (q \wedge r)]$

3 Truth table

p	q	r	$\sim p$	$\sim r$	$p \wedge \sim r$	$\sim p \vee q$	$(p \wedge \sim r) \Rightarrow (\sim p \vee q)$
T	T	T	F	F	F	T	T
T	T	F	F	T	T	T	T
T	F	T	F	F	F	F	T
T	F	F	F	T	T	F	F
F	T	T	T	F	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	T	F	T	T

Since, $(p \wedge \sim r) \Rightarrow (\sim p \vee q)$ is F.
Then, $p = T, q = F, r = F$

4 Consider, $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$$\begin{aligned}
&\equiv [(p \wedge \sim q) \vee q] \vee (\sim p \wedge q)\\
&\equiv [(p \vee q) \wedge (\sim q \vee q)] \vee (\sim p \wedge q)\\
&\equiv [(p \vee q) \wedge t] \vee (\sim p \wedge q)\\
&\equiv (p \vee q) \vee (\sim p \wedge q)\\
&\equiv (p \vee q \vee \sim p) \wedge (p \vee q \vee q)\\
&\equiv (q \vee t) \wedge (p \vee q) \equiv t \wedge (p \vee q) \equiv p \vee q
\end{aligned}$$

5 Truth Table

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (p \rightarrow q)$	$(p \rightarrow q) \vee (p \rightarrow q)$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

So, only $(p \rightarrow q) \vee (q \rightarrow p)$ is a tautology.

6 Since, p is true and q is false $\Rightarrow \sim p$ is false and $\sim q$ is true.

$p \leftrightarrow q$ is F	[since, p is true, q is false]
$\sim p \leftrightarrow q$ is T	[since, $\sim p$ is false, q is false]
$\sim q \leftrightarrow p$ is T	[since, $\sim q$ is true, p is true]
$\sim p \leftrightarrow \sim q$ is F	[since, $\sim p$ is false, $\sim q$ is true]

7

A	B	$A \vee B$	$A \wedge B$	$A \vee (A \vee B)$	$A \vee (A \wedge B)$
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	F	T	F
F	F	F	F	F	F

$A \rightarrow B$	$A \wedge (A \rightarrow B)$	$A \wedge (A \rightarrow B) \rightarrow B$	$B \rightarrow (A \wedge (A \rightarrow B))$
T	T	T	T
F	F	T	T
T	F	T	F
T	F	T	T

Since, the truth value of all the elements in the column $A \wedge (A \rightarrow B) \rightarrow B$
So, $A \wedge (A \rightarrow B) \rightarrow B$ is tautology.

8

p	q	$\sim p$	$\sim q$	$(p \wedge q)$	$\sim(p \wedge q)$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	F	T

$\sim p \vee \sim q$	$q \wedge p$	$p \Leftrightarrow q$	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Leftrightarrow q) \vee (q \Rightarrow p)$
F	T	T	T	T	T
T	F	F	F	T	T
T	F	F	T	F	T
T	F	T	T	F	T

It is clear from the table that false statement is $p \Leftrightarrow q \equiv (p \Rightarrow q) \vee (q \Rightarrow p)$

Hence, it is clear from the table that $p \Leftrightarrow q$ and $(p \Rightarrow q) \vee (q \Rightarrow p)$ is not logically equilibrium.

9

p	q	$p \Leftrightarrow q$	$\sim(p \Leftrightarrow q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(p \Leftrightarrow q) \Leftrightarrow \sim p \vee \sim q$
F	F	F	T
F	T	T	T
T	F	T	T
T	T	T	F

Last column shows that result is neither a tautology nor a contradiction.

10 The truth tables of $p \rightarrow q$ and $\sim p \vee q$ are given below

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Clearly, truth tables of $p \rightarrow q$ and $\sim p \vee q$ are same.
So, $p \rightarrow q$ is logically equivalent to $\sim p \vee q$.
Hence, option (a) is correct.
If the truth value of p, q, r are T, F, T respectively, then the truth values of $p \vee q$ and $q \vee r$ are each equal to T. Therefore, the truth value of $(p \vee q) \wedge (q \vee r)$ is T.
Hence, option (b) is correct.
We know that, $\sim(p \vee q \vee r) \equiv (\sim p \wedge \sim q \wedge \sim r)$
So, option (c) is correct.
If p is true and q is false, then $p \vee q$ is true. Consequently $\sim(p \vee q)$ is false and hence $p \wedge \sim(p \vee q)$ is false.
Hence, option (d) is wrong.

11

p	q	r	$p \wedge q$	$p \wedge r$	$(p \wedge q) \wedge (q \wedge r)$
T	F	F	F	F	F
T	F	T	F	T	F
T	T	F	T	F	F
T	T	T	T	T	T
F	F	F	F	F	F
F	F	T	F	F	F
F	T	F	F	F	F
F	T	T	F	F	F

12.

p	q	$\sim p$	$\sim p \wedge q$	$p \Rightarrow q$	$q \Rightarrow p$	$\sim(p \Rightarrow q)$	$\sim(q \Rightarrow p)$
T	T	F	F	T	T	F	F
T	F	F	F	F	T	T	F
F	T	T	T	T	F	F	T
F	F	T	F	T	T	F	F

It is clear from the above table that columns 4 and 8 are equal.
Hence, $\sim p \wedge q$ is equivalent to $\sim(q \Rightarrow p)$.

13

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \vee \sim q$	$p \vee \sim p$	$\sim[p \vee \sim q]$
T	T	F	F	F	T	T	F
T	F	F	T	F	T	T	F
F	T	T	F	T	F	T	T
F	F	T	T	F	T	T	F

So, $\sim p \wedge q \equiv \sim[p \vee \sim q]$ and $p \vee \sim p$ is a tautology.

14 **Statement II** $(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\equiv (p \to q) \leftrightarrow (p \to q)$

which is always true, so Statement II is true.

Statement I $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$\equiv p \wedge \sim q \wedge \sim p \wedge q \equiv p \wedge \sim p \wedge \sim q \wedge q \equiv f \wedge f \equiv f$

Hence, it is a fallacy statement. So, Statement I is true.

Alternate Method

Statement II $(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\sim q \to \sim p$ is contrapositive of $p \to q$

Hence, $(p \to q) \leftrightarrow (p \to q)$ will be a tautology.

Statement I $(p \wedge \sim q) \wedge (\sim p \wedge q)$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Hence, it is a fallacy.

15

A	B	$A \vee B$	$B \to A$	$A \wedge B$	$\sim A$	$\sim A \vee B$	$A \to (A \vee B)$
T	T	T	T	T	F	T	T
T	F	T	T	F	F	F	T
F	T	T	F	F	T	T	T
F	F	F	T	F	T	T	T